

# Wormhole solutions in the presence of nonlinear Maxwell field

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Generalizing of the Maxwell field to nonlinear electrodynamics theories, we look for the magnetic solutions. In initial approximation these models give the usual linear electrodynamics. We consider a suitable metric and investigate the properties of the solutions. Also, we use the cut-and-paste method to construct wormhole structure. We generalize the static solutions to rotating spacetime and obtain conserved quantities.

## I. INTRODUCTION

A wormhole can be defined as a tunnel which can joint two universes [1]. Since General Relativity does not preclude the existence of (traversable) wormholes, a large number of papers have been written which clarify, support, or contradict much of the research about wormholes. Morris and Thorne [1] have shown that in order to construct a traversable wormhole, one needs to have extraordinary material, denoted as exotic matter. Exotic matter can guarantee the flare-out condition of the wormhole at its throat.

Unlike the classical form of matter [2], it is believed that the exotic matter violates the well-known energy conditions such as the null energy conditions (NEC), weak energy conditions (WEC), strong energy conditions (SEC) and dominant energy conditions (DEC). One of the open questions about the exotic matter is that if it can be formed in macroscopic quantities or not. We should note that these energy conditions are violated by certain states of quantum fields, amongst which we may refer to the Casimir energy, Hawking evaporation, and vacuum polarization [3]. Furthermore, it has been shown that one of the effective causes of the (late time) cosmic acceleration is an exotic fluid [4]. Hence, it will be motivated to study wormhole solutions, at least geometrically.

Traversable wormholes in the Dvali-Gabadadze-Porrati theory with cylindrical symmetry has been studied in Ref. [5]. Higher dimensional Lorentzian wormholes have been analyzed by several authors [7]. Moreover, wormhole solutions of higher derivative gravity with linear and nonlinear

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Maxwell fields have been considered in [8]. For other kinds of wormhole solutions, we refer the reader to Refs. [6, 8] and references therein.

Many authors have extensively considered the nonlinear electrodynamics and used their results to explain some physical phenomena [9–13]. A charged system whose performance cannot be described by the linear equations may be characterized with nonlinear electrodynamics. From mathematical point of view, since Maxwell equations originated from the empirical nature, we can consider a general nonlinear theory of electrodynamics and state that the Maxwell fields, are only approximations of nonlinear electrodynamics, which the approximation breaks down for the small distances. From physical viewpoint, generalizations of Maxwell theory to nonlinear electrodynamics were introduced to eliminate infinite quantities in theoretical analysis of the electrodynamics [11]. In addition, one may find some various limitations of the linear electrodynamics in Ref. [14].

Recently, we take into account new classes of nonlinear electrodynamics, such as Born-Infeld (BI) like [13] and power-Maxwell invariant (PMI) [12] nonlinear electrodynamics, in order to obtain new analytical solutions in Einstein and higher derivative gravity.

Motivated above, in this paper we look for the analytical magnetic horizonless solutions of Einstein gravity with nonlinear Maxwell source. Properties of the solutions will be investigated.

## II. FIELD EQUATIONS AND WORMHOLE SOLUTIONS:

The field equations of Einstein gravity with an arbitrary  $U(1)$  gauge field as a source, may be written as

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{1}{2}g_{\mu\nu}L(\mathcal{F}) - 2L_{\mathcal{F}}F_{\mu\lambda}F_{\nu}^{\lambda}, \quad (1)$$

$$\partial_{\mu}(\sqrt{-g}L_{\mathcal{F}}F^{\mu\nu}) = 0, \quad (2)$$

where  $G_{\mu\nu}$  is the Einstein tensor,  $\Lambda = -3/2l^2$  denotes the four dimensional negative cosmological constant,  $L(\mathcal{F})$  is an arbitrary function of the closed 2-form Maxwell invariant  $\mathcal{F} = F_{\mu\nu}F^{\mu\nu}$  and  $L_{\mathcal{F}} = \frac{dL(\mathcal{F})}{d\mathcal{F}}$ .

In addition to PMI and BI theories, in this paper, we take into account the recently proposed BI-like models [13], which we called them as Exponential form of nonlinear electrodynamics theory

(ENE) and Logarithmic form of nonlinear electrodynamics theory (LNE), whose Lagrangians are

$$L(\mathcal{F}) = \begin{cases} (-\mathcal{F})^s, & PMI \\ 4\beta^2 \left(1 - \sqrt{1 + \frac{\mathcal{F}}{2\beta^2}}\right), & BI \\ \beta^2 \left(\exp(-\frac{\mathcal{F}}{\beta^2}) - 1\right), & ENE \\ -8\beta^2 \ln \left(1 + \frac{\mathcal{F}}{8\beta^2}\right), & LNE \end{cases}, \quad (3)$$

where  $s$  and  $\beta$  are two nonlinearity parameters. Expanding the mentioned Lagrangians near the linear Maxwell case ( $s \rightarrow 1$  and  $\beta \rightarrow \infty$ ), one can obtain

$$L(\mathcal{F}) \rightarrow L_{Max} + \begin{cases} -\mathcal{F} \ln(-\mathcal{F})(s-1) + O(s-1)^2, & PMI \\ +\frac{\chi\mathcal{F}^2}{16\beta^2} + O\left(\frac{\mathcal{F}^3}{\beta^4}\right), & others \end{cases}, \quad (4)$$

where Maxwell Lagrangian  $L_{Max} = -\mathcal{F}$  and  $\chi = 1, 2$  and  $8$  for LNE, BI and ENE branches, respectively.

Investigation of the effects of the higher derivative corrections to the gauge field seems to be an interesting phenomenon. These nonlinear electrodynamics sources have different effects on the physical properties of the solutions. For example in black hole framework, these nonlinearities may change the electric potential, temperature, horizon geometry, energy density distribution and also asymptotic behavior of the solutions. In what follows, we study the effects of nonlinearity on the magnetic solutions.

Motivated by the fact that we are looking for the horizonless magnetic solution (instead of electric one), one can start with the following 4-dimensional spacetime

$$ds^2 = -\frac{r^2}{l^2}dt^2 + \frac{dr^2}{f(r)} + \Upsilon^2 l^2 f(r) d\theta^2 + r^2 d\phi^2, \quad (5)$$

where  $\Upsilon$  is a constant and will be fixed later. We should note that, because of the periodic nature of  $\theta$ , one can obtain the presented metric (5) with *local* transformations  $t \rightarrow il\Upsilon\theta$  and  $\theta \rightarrow it/l$  in the horizon-flat Schwarzschild metric,  $ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2(d\theta^2 + d\phi^2)$ . In other words, metric (5) may be *locally* mapped to Schwarzschild spacetime, but not *globally*. Considering the mentioned local transformation, one can find that the nonzero component of the gauge potential is  $A_\theta$

$$A_\mu = h(r)\delta_\mu^\theta, \quad (6)$$

where  $h(r)$  is an arbitrary function of  $r$ . Using Eq. (2) with the metric (5), we find  $h(r) = \int E(r)dr$

in which

$$E(r) = \begin{cases} \frac{2ql^2\Upsilon^2}{r^{2/(2s-1)}}, & PMI \\ \frac{2ql^2\Upsilon^2}{r^2\sqrt{1+\frac{4q^2l^4\Upsilon^2}{\beta^2r^4}}}, & BI \\ \frac{2ql^2\Upsilon^2}{r^2}\exp\left(-\frac{L_W}{2}\right), & ENE \\ \frac{\beta^2r^2(\Gamma-1)}{ql^2}, & LNE \end{cases}, \quad (7)$$

where  $L_W = \text{LambertW}(X)$ ,  $X = \frac{16l^4q^2\Upsilon^2}{\beta^2r^4}$  and  $\Gamma = \sqrt{1 + \frac{X}{4}}$  and therefore the nonzero component of electromagnetic field tensor is

$$F_{r\theta} = E(r). \quad (8)$$

We should note that the physical gauge potential should vanish for large values of  $r$ . This condition is satisfied for  $1/2 < s < 3/2$  and arbitrary  $\beta$  (the mentioned constrain for  $s$  is used throughout the rest of the paper). Now, one can expand Eq. (7) to obtain the leading nonlinearity correction of Maxwell field

$$E(r)|_{\text{near the linear case}} = \frac{2ql^2\Upsilon^2}{r^2} + \begin{cases} \frac{8ql^2\Upsilon^2\ln(r)}{r^2}(s-1) + O(s-1)^2, & PMI \\ -\frac{2\chi q^3l^6\Upsilon^4}{\beta^2r^6} + O(\frac{1}{\beta^4}), & others \end{cases}. \quad (9)$$

In order to examine the effect of nonlinearity on the electromagnetic field, we plot Figs. 1 and 2. Figure 1 shows that when we reduce the nonlinearity  $s$ , the electromagnetic field of the PMI branch diverges for  $r \rightarrow 0$  more rapidly and for large distances it goes to zero more quickly. Figure 2 shows that for all BI-like branches, the electromagnetic field (the same behavior as in Maxwell case) vanishes for large  $r$ . Near the origin the electromagnetic field of BI and LNE branches have finite values, but for ENE branch, it diverges. Comparing this divergency with Maxwell one, we find that ENE divergency is more slowly.

Taking into account the electromagnetic field tensor, we are in a position to find the function  $f(r)$ . In order to obtain it, one may use any components of Eq. (1). We simplify the components of Eq. (1) and find that the nonzero independent components of Eq. (1) are

$$f''(r) + \frac{2f'(r)}{r} + 2\Lambda + \Delta_1(r) = 0, \quad (10)$$

$$f'(r) + \frac{f(r)}{r} + \Lambda r + \Delta_2(r) = 0, \quad (11)$$

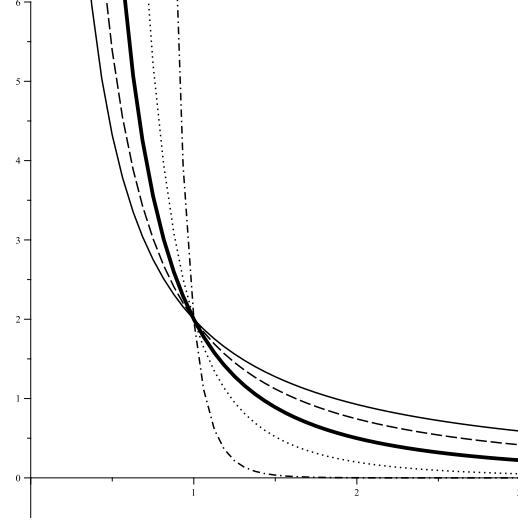


FIG. 1:  $E(r)$  versus  $r$  for  $q = 1$ ,  $\Upsilon = 1$ ,  $l = 1$  and  $s = 1.4$  (solid line),  $s = 1.2$  (dashed line),  $s = 1$  "Maxwell field" (bold line),  $s = 0.8$  (dotted line) and  $s = 0.6$  (dash-dotted line)

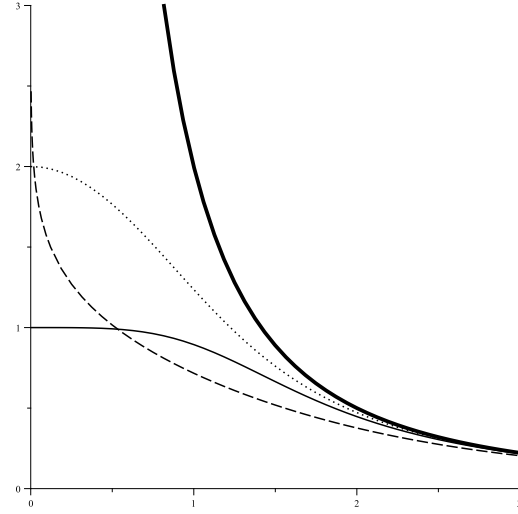


FIG. 2:  $E(r)$  versus  $r$  for  $q = 1$ ,  $\Upsilon = 1$ ,  $l = 1$  and  $\beta = 1$ . BI (solid line), ENE (dashed line), LNE (dotted line) and Maxwell field (bold line)

with

$$\Delta_1(r) = \begin{cases} -\left(\frac{2h'^2(r)}{\Upsilon^2}\right)^s, & PMI \\ 4\beta^2 \left[ \sqrt{1 - \frac{h'^2(r)}{\beta^2 \Upsilon^2}} - 1 \right], & BI \\ \beta^2 \left[ 1 - \exp\left(\frac{2h'^2(r)}{\beta^2 \Upsilon^2}\right) \right], & ENE \\ 8\beta^2 \ln \left[ 1 - \left(\frac{h'(r)}{2\beta \Upsilon}\right)^2 \right], & LNE \end{cases}, \quad (12)$$

$$\Delta_2(r) = \begin{cases} \frac{r}{2}(2s-1) \left( \frac{2h'^2(r)}{\Upsilon^2} \right)^s, & PMI \\ 2r\beta^2 \left[ \left( 1 - \frac{h'^2(r)}{\beta^2 \Upsilon^2} \right)^{-1/2} - 1 \right], & BI \\ \frac{r\beta^2}{2} \left\{ 1 - \left[ 1 - \left( \frac{2h'(r)}{\beta \Upsilon} \right)^2 \right] \exp \left( \frac{2h'^2(r)}{\beta^2 \Upsilon^2} \right) \right\}, & ENE \\ 4r\beta^2 \left\{ \ln \left[ 1 - \left( \frac{h'(r)}{2\beta \Upsilon} \right)^2 \right] - \frac{2}{1 - \left( \frac{2\beta \Upsilon}{h'(r)} \right)^2} \right\}, & LNE \end{cases}, \quad (13)$$

where prime and double prime are first and second derivatives with respect to  $r$ , respectively.

After some cumbersome manipulation, the solutions of Eqs. (10) and (11) can be written as

$$f(r) = \frac{-2m}{r} - \frac{\Lambda r^2}{3} + \begin{cases} -\frac{r^2(2s-1)^2}{2(2s-3)} \left( \frac{8q^2 l^4 \Upsilon^2}{r^{4/(2s-1)}} \right)^s, & PMI \\ \frac{2\beta^2 r^2}{3} - \frac{2\beta^2}{r} \int \sqrt{1 + \frac{4q^2 l^4 \Upsilon^2}{\beta^2 r^4}} r^2 dr, & BI \\ -\frac{\beta^2 r^2}{6} + \frac{2\beta q l^2 \Upsilon}{r} \int \left( \frac{1}{\sqrt{LW}} - \sqrt{LW} \right) dr, & ENE \\ +\frac{8\beta^2 r^2}{3} - \frac{4\beta^2}{r} \int r^2 \ln \left( \frac{\beta^2 r^4 (\Gamma-1)}{2q^2 \Upsilon^2 l^4} \right) dr - \frac{16l^4 q^2 \Upsilon^2}{r} \int \frac{dr}{r^2 (\Gamma-1)}, & LNE \end{cases}, \quad (14)$$

where  $m$  is the integration constant which is related to the mass parameter. In order to investigate the effect of nonlinearity on the metric function, simplistically, we expand  $f(r)$  for  $s \rightarrow 1$  for PMI and  $\beta \rightarrow \infty$  for other branches. After some manipulation, we obtain

$$f(r) = f_{Max}(r) + \begin{cases} \frac{4q^2 l^4 \Upsilon^2 [6 + \ln(8q^2 \Upsilon^2 l^4 r^4)]}{r^2} (s-1) + O(s-1)^2, & PMI \\ -\frac{2\chi q^4 l^8 \Upsilon^4}{5\beta^2 r^6} + O\left(\frac{1}{\beta^4}\right), & others \end{cases}, \quad (15)$$

where  $f_{Max}(r)$  is the magnetic solution of Einstein-Maxwell gravity

$$f_{Max}(r) = \frac{-2m}{r} - \frac{\Lambda r^2}{3} + \frac{4q^2 l^4 \Upsilon^2}{r^2}, \quad (16)$$

and the second term on the right hand side of Eq. (15) is the leading nonlinearity correction to the Einstein-Maxwell gravity solution.

### A. Properties of the solutions

At first step, we should note that the presented solutions are asymptotically anti-de Sitter (adS) and they reduce to asymptotically adS Einstein-Maxwell solutions for  $s \rightarrow 1$  (PMI branch) or  $\beta \rightarrow \infty$  (other branches).

The second step should be devoted to singularities and hence we should calculate the curvature invariants. One can show that for the metric (5), the Kretschmann and Ricci scalars are

$$R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} = f''^2(r) + \frac{4f'^2(r)}{r^2} + \frac{4f^2(r)}{r^4}, \quad (17)$$

$$R = -f''(r) - \frac{4f'(r)}{r} - \frac{2f(r)}{r^2}. \quad (18)$$

Inserting Eq. (14) into the Eqs. (17) and (18), and using numerical calculations, one can show that the Ricci and Kretschmann scalars diverge at  $r = 0$ , are finite for  $r > 0$  and for  $r \rightarrow \infty$  one obtains

$$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}|_{Large\ r} = \frac{8\Lambda^2}{3} + \begin{cases} O\left(\frac{1}{r^\xi}\right) & PMI \\ O\left(\frac{1}{r^6}\right), & others \end{cases}, \quad (19)$$

$$R|_{Large\ r} = 4\Lambda + \begin{cases} O\left(\frac{1}{r^\xi}\right) & PMI \\ O\left(\frac{1}{r^8}\right), & others \end{cases}, \quad (20)$$

$$\xi \in (3, \infty),$$

which confirms that the asymptotic behavior of the solutions is adS. Considering the divergency of the Ricci and Kretschmann scalars at the origin, one may think that there is a curvature singularity located at  $r = 0$ . This singularity will be naked if the function  $f(r)$  has no real root (singularity is not covered with a horizon) and we are not interested in it. Therefore, we consider the case in which the function  $f(r)$  has at least a non-extreme positive real root. It is notable that the function  $f(r)$  is negative for  $r = r_+ - \epsilon$  ( $\epsilon$  is an infinitesimal number), and positive for  $r > r_+$  where  $r_+$  is the largest positive real root of  $f(r) = 0$ . Negativity of the function  $f(r)$  leads to an apparent change of metric signature and it forces us to consider  $r_+ \leq r < \infty$ . We should state that although the metric function  $f(r)$  vanishes at  $r = r_+$ , but we have  $f'(r = r_+) \neq 0$  ( $f'(r = r_+) > 0$ ). In addition, there is no curvature singularity in the range  $r_+ \leq r < \infty$ . Following the procedure of Ref. [15], one may find that there is a conic singularity at  $r = r_+$ .

Removing this conical singularity with  $\Upsilon = 1/[lf'(r_+)]$  [15], we desire to interpret the obtained solutions as wormholes. In order to construct wormholes from the gluing, one requires to use the cut-and-paste prescription [6]. In this method, we take into account two geodesically incomplete copies of the solutions (removing from each copy the forbidden region  $\Omega$ ) with two copies of the boundaries  $\partial\Omega$ , where

$$\Omega \equiv \{r \mid r < r_+\}, \quad (21)$$

$$\partial\Omega \equiv \{r \mid r = r_+\}. \quad (22)$$

Now, we identify two copies of the mentioned boundaries to build a geodesically complete manifold. This cut-and-paste method constructs a wormhole with a throat at  $r = r_+$ . In order to confirm this claim, we should check the so-called flare-out condition at the throat. To do this, one can consider a 2-dimensional submanifold of the metric (5),  $ds_{2-\text{dim}}^2$ , (with constant  $t$  and  $\theta$ ) and embed it in a

3-dimensional Euclidean flat space in cylindrical coordinates,  $ds_{3-\text{dim}}^2$ , where

$$ds_{2-\text{dim}}^2 = \frac{dr^2}{f(r)} + r^2 d\phi^2, \quad (23)$$

$$ds_{3-\text{dim}}^2 = dr^2 + r^2 d\phi^2 + dz^2. \quad (24)$$

Regarding the surface  $z = z(r)$ , we obtain

$$\left. \frac{dr}{dz} \right|_{r=r_+} = \left. \sqrt{\frac{f(r)}{1-f(r)}} \right|_{r=r_+} = 0, \quad (25)$$

$$\left. \frac{d^2 r}{dz^2} \right|_{r=r_+} = \left. \frac{f'}{2[1-f]^2} \right|_{r=r_+} = \frac{1}{2} f'(r=r_+) > 0, \quad (26)$$

which shows that the flare-out condition may be satisfied for the surface  $z = z(r)$  and therefore  $r = r_+$  is the radius of the wormhole throat.

Now, we should discuss the energy conditions for the wormhole solutions. We should note that, traversable wormhole may exist with exotic matter which violates the null energy condition [1]. We use the following orthonormal contravariant (hatted) basis to simplify the mathematics and physical interpretations

$$\mathbf{e}_{\hat{t}} = \frac{l}{r} \frac{\partial}{\partial t}, \quad \mathbf{e}_{\hat{r}} = f^{1/2} \frac{\partial}{\partial r}, \quad \mathbf{e}_{\hat{\theta}} = \frac{1}{\Upsilon l f^{1/2}} \frac{\partial}{\partial \theta}, \quad \mathbf{e}_{\hat{\phi}} = r^{-1} \frac{\partial}{\partial \phi}. \quad (27)$$

Using the mentioned basis, we can obtain

$$T_{\hat{t}\hat{t}} = -T_{\hat{\phi}\hat{\phi}} = \begin{cases} -\frac{1}{2} \left( \frac{8\Upsilon^2 q^2 l^4}{r^{4/(2s-1)}} \right)^s, & PMI \\ 2\beta^2 (\Gamma^{-1} - 1), & BI \\ \frac{\beta^2}{2} \left( 1 - \sqrt{\frac{X}{L_W}} \right), & ENE \\ 4\beta^2 \ln \left( \frac{8(\Gamma-1)}{X} \right), & LNE \end{cases}, \quad (28)$$

$$T_{\hat{r}\hat{r}} = -T_{\hat{\theta}\hat{\theta}} = \begin{cases} -\frac{(2s-1)}{2} \left( \frac{8\Upsilon^2 q^2 l^4}{r^{4/(2s-1)}} \right)^s, & PMI \\ 2\beta^2 (1 - \Gamma), & BI \\ \frac{\beta^2}{2} \left( \sqrt{\frac{X}{L_W}} - \sqrt{X L_W} - 1 \right), & ENE \\ \beta^2 \left[ 8 - 4 \ln \left( \frac{8(\Gamma-1)}{X} \right) - \frac{X}{(\Gamma-1)} \right], & LNE \end{cases}, \quad (29)$$

and therefore

$$T_{\hat{t}\hat{t}} < 0$$

$$T_{\hat{t}\hat{t}} + T_{\hat{r}\hat{r}} < 0,$$

which shows that all the energy conditions are violated as well (see Fig. 3 for more clarification).



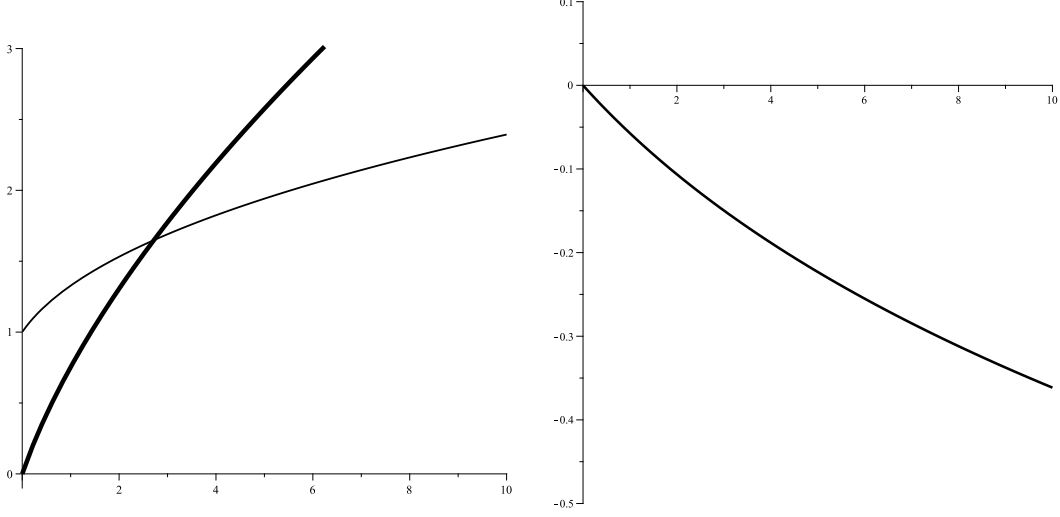


FIG. 3: **left figure:**  $\sqrt{\frac{L_W}{X}}$  (solid line) and  $\sqrt{X L_W}$  (bold line) versus  $X$ . **right figure:**  $\ln\left(\frac{8(\Gamma-1)}{X}\right)$  versus  $X$ .

At the end of this section, we desire to study the effects of the nonlinearity on energy density of the spacetime. At the start, we can expand  $T_{\hat{t}\hat{t}}$  near the linear case to obtain

$$T_{\hat{t}\hat{t}} = T_{\hat{t}\hat{t}}|_{Maxwell} + \begin{cases} -\frac{4q^2 l^4 \Upsilon^2 \ln(8q^2 \Upsilon^2 l^4 r^4)}{r^4} (s-1) + O(s-1)^2, & PMI \\ +\frac{6\chi q^4 l^8 \Upsilon^4}{\beta^2 r^8} + O(\frac{1}{\beta^4}), & others \end{cases}, \quad (30)$$

where  $T_{\hat{t}\hat{t}}|_{Maxwell} = \frac{-4\Upsilon^2 q^2 l^4}{r^4}$  and the second term on the right hand side of Eq. (30) is the leading nonlinearity correction to the energy density of the Einstein-Maxwell theory. In addition, we plot the energy density  $T_{\hat{t}\hat{t}}$  versus  $r$  for different values of nonlinearity parameter  $s$  and also various branches of BI-like fields. Figures 4 and 5, show that for the arbitrary choices of  $r$  the energy density is negative. Furthermore, Fig. 4 shows that the nonlinearity parameter,  $s$ , has effects on the behavior of the energy density and when we reduce  $s$ , both divergency of energy density near the origin and its vanishing for large values of distance occur more rapidly. Moreover, considering Fig. 5, one can find that  $T_{\hat{t}\hat{t}}$  has a finite value for an arbitrary distance in BI branch and it diverges near the origin for other branches. It is notable that, near the origin, the divergency of LNE branch is stronger than ENE branch. Also, for BI-like branches, the nonlinearity reduces the strength of energy density divergency.

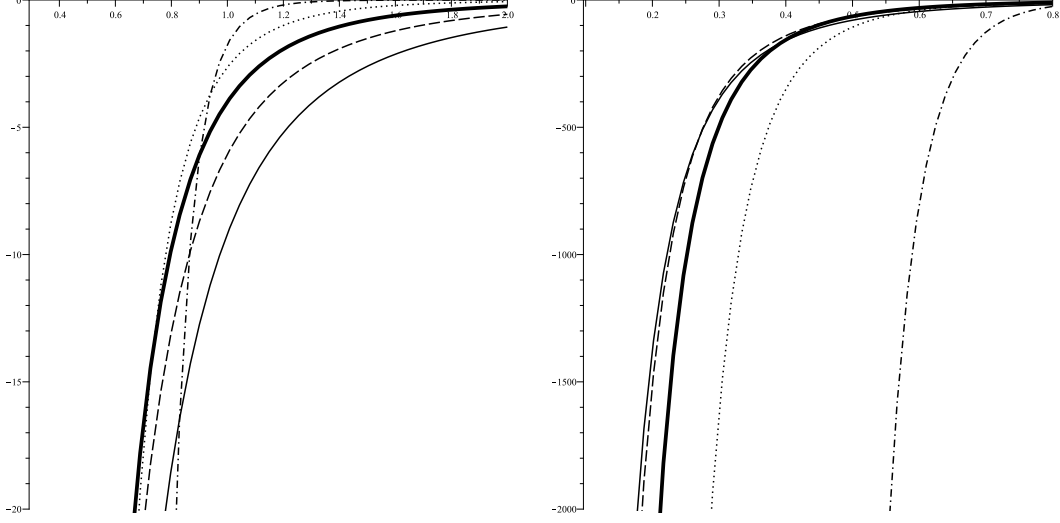


FIG. 4:  $T_{\hat{t}\hat{t}}$  versus  $r$  for  $q = 1$ ,  $\Upsilon = 1$ ,  $l = 1$  and  $s = 1.4$  (solid line),  $s = 1.2$  (dashed line),  $s = 1$  "Maxwell field" (bold line),  $s = 0.8$  (dotted line) and  $s = 0.6$  (dash-dotted line) "different scales"

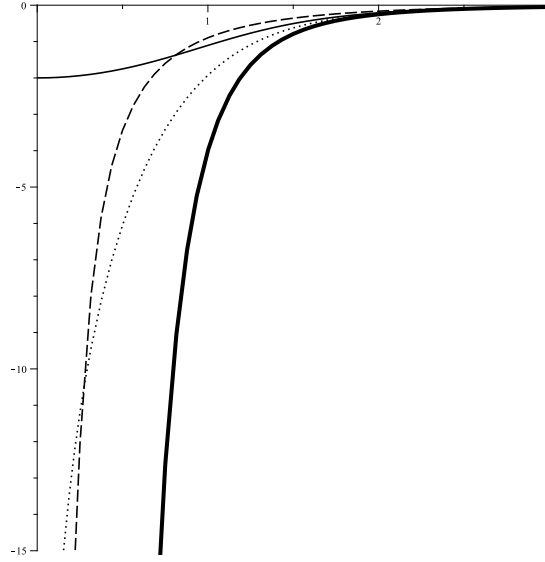


FIG. 5:  $T_{\hat{t}\hat{t}}$  versus  $r$  for  $q = 1$ ,  $\Upsilon = 1$ ,  $l = 1$  and  $\beta = 1$ . BI (solid line), ENE (dashed line), LNE (dotted line) and Maxwell field (bold line)

## B. Rotating solutions

In this section, we want to add angular momentum to the static spacetime (5). To do this, one can use the following rotation boost in the  $t - \theta$  plane

$$t \mapsto \Xi t - a\theta, \quad \theta \mapsto \Xi\theta - \frac{a}{l^2}t, \quad (31)$$

where  $\Xi = \sqrt{1 + a^2/l^2}$  and  $a$  is a rotation parameter. Taking into account Eq. (31) and applying it to static metric (5), one obtains

$$ds^2 = -\frac{r^2}{l^2} (\Xi dt - a d\theta)^2 + \frac{dr^2}{f(r)} + \Upsilon^2 l^2 f(r) \left( \frac{a}{l^2} dt - \Xi d\theta \right)^2 + r^2 d\phi^2, \quad (32)$$

where  $f(r)$  is the same as  $f(r)$  given in Eq. (14). It is notable that one can obtain the presented metric (32) with *local* transformations  $t \rightarrow il\Upsilon (at/l^2 - \Xi\theta)$  and  $\theta \rightarrow i(\Xi t - a\theta)/l$  in the horizon-flat Schwarzschild metric,  $ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\theta^2 + r^2 d\phi^2$ . Thus, the nonzero components of the gauge potential are  $A_\theta$  and  $A_t$

$$A_\mu = h(r) \left( \Xi \delta_\mu^\theta - \frac{a}{l^2} \delta_\mu^t \right), \quad (33)$$

where  $h(r)$  is the same as in the static case. Furthermore, the nonzero components of electromagnetic field tensor are given by

$$F_{tr} = \frac{a}{\Xi l^2} F_{r\theta} = \frac{a}{l^2} E(r). \quad (34)$$

As we mentioned before, the periodic nature of  $\theta$  helps us to conclude that the transformation (31) is not a proper coordinate transformation on the entire manifold and therefore the metrics (5) and (32) are distinct [16]. In addition, it is desired to note that rotating solutions have no horizon and curvature singularity. Moreover, it is worth noting that besides the magnetic field along the  $\theta$  coordinate, there is also a radial electric field ( $F_{tr} \neq 0$ ) and therefore, unlike the static case, the rotating wormhole has a nonzero electric charge which is proportional to the rotation parameter.

### C. Conserved Quantities

Here we desire to calculate finite conserved quantities. In order to obtain a finite value for these quantities, we can use the counterterm method inspired by the concept of (AdS/CFT) correspondence [17]. It has been shown that for asymptotically AdS solutions the finite energy momentum tensor is

$$T^{ab} = \frac{1}{8\pi} \left( K^{ab} - K \gamma^{ab} - \frac{2\gamma^{ab}}{l} \right), \quad (35)$$

where  $K$  is the trace of the extrinsic curvature  $K^{ab}$  and  $\gamma^{ab}$  is the induced metric of the boundary. Taking into account the Killing vector field  $\xi$ , one may obtain the quasilocal conserved quantities in the following form

$$\mathcal{Q}(\xi) = \int_B d^2\varphi \sqrt{\sigma} T_{ab} n^a \xi^b, \quad (36)$$

where  $\sigma$  is the determinant of the boundary metric in ADM (Arnowitt-Deser-Misner) form  $\sigma_{ij}$ , and  $n^a$  is the timelike unit vector normal to the boundary  $\mathcal{B}$ . Considering two Killing vectors  $\xi = \partial/\partial t$  and  $\zeta = \partial/\partial \theta$ , we can find their associated conserved charges which are mass and angular momentum

$$M = 4\pi^2 [3(\Xi^2 - 1) + 1] \Upsilon m, \quad (37)$$

$$J = 12\pi^2 \Upsilon m \Xi a, \quad (38)$$

where the former equation confirms that  $a$  is the rotational parameter.

Finally, we are in a position to discuss the electric charge. In order to compute it, we need a nonzero radial electric field  $F_{tr}$  and therefore one expects vanishing  $F_{tr}$  (static case) leads to zero electric charge. Taking into account the Gauss's law for the rotating solutions and computing the flux of the electric field at infinity, one can find

$$Q = \begin{cases} \frac{2^{3s+1}\pi^2\Upsilon s}{4l} \left( \frac{(2s-1)q}{(3-2s)l} \right)^{2s-1} a, & PMI \\ \frac{4\pi^2\Upsilon q}{l^2} a, & others \end{cases}, \quad (39)$$

which confirms that the static wormholes do not have electric charge.

### III. CLOSING REMARKS

In this paper, we took into account a class of magnetic Einsteinian solutions in the presence of nonlinear source. The magnetic spacetime which we used in this paper, may be obtained from the horizon-flat Schwarzschild metric with *local* transformations  $t \rightarrow il\Upsilon\theta$  and  $\theta \rightarrow it/l$ . It is notable that because of the periodic nature of  $\theta$ , the mentioned transformations cannot be global.

We considered four forms of nonlinear electrodynamics, namely PMI, BI, ENE and LNE theories, whose asymptotic behavior leads to Maxwell theory. We investigated the effect of nonlinearity parameter on the electromagnetic field and found that for PMI branch, if one reduces the nonlinearity parameter  $s$ , then the electromagnetic field diverges near the origin more rapidly and for large distances it goes to zero more quickly. In addition, we found that for all BI-like branches, the behavior of the electromagnetic field is the same as Maxwell case for large values of distance, but near the origin, the electromagnetic field of the BI and LNE branches is finite and it diverges for the ENE branch. It is interesting to note that the divergency of the ENE branch has less strength in comparison to the the Maxwell field.

Then, we obtained the metric function for all branches and found that they reduce to asymptotically adS Einstein-Maxwell solutions for  $s \rightarrow 1$  (PMI branch) or  $\beta \rightarrow \infty$  (other branches). We also expanded the metric function near the linear Maxwell field and calculated the curvature scalars for large  $r$  to find that obtained solutions are asymptotically anti-de Sitter (adS). Taking into account the presented metric, one can find that the function  $f(r)$  cannot be negative since its negativity leads to an apparent change of metric signature. This limitation forced us to consider  $r_+ \leq r < \infty$ . Using numerical calculations, one can find that there is no curvature singularity in the range  $r_+ \leq r < \infty$ , but one may find a conic singularity at  $r = r_+$ .

After that, we removed the mentioned conic singularity and used the cut-and-paste prescription to construct a wormhole from the gluing and then we checked the so-called flare-out condition at the throat  $r = r_+$ . Since it has been stated before, that traversable wormhole may exist with exotic matter [1], we investigated the energy conditions for the obtained wormhole solutions and found that the energy conditions are violated.

We also studied the effects of nonlinearity parameter on the energy density. We found that when we reduce  $s$ , both divergency of energy density near the origin and its vanishing for large values of distance occur more rapidly. Moreover, one can find that energy density has a finite value for an arbitrary distance only in BI branch and it diverges near the origin for other BI-like branches. It is notable that, near the origin, the divergency of LNE branch is stronger than ENE branch.

We generalized the static solutions to rotating ones and obtained the conserved quantities. We found that, unlike the static case, for the spinning spacetime, the wormhole has a net electric charge density. We also found that in spite of the fact that the mentioned nonlinear theories change the properties of the solutions significantly, but they do not have any effect on mass and angular momentum.

### Acknowledgments

We are indebted to H. Mohammadpour for reading the manuscript. We also wish to thank Shiraz University Research Council. This work has been supported financially by Research Institute for Astronomy & Astrophysics of Maragha (RIAAM), Iran.

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